MHD Stability for a Class of Tokamak Equilibria with Fixed Boundaries *

W. Kerner

Max-Planck-Institut für Plasmaphysik, Garching bei München, Federal Republic of Germany

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The stability behavior with respect to internal modes is discussed for a class of tokamak equilibria with non-circular cross-sections and essentially flat current profiles. The stability analysis is done by computer both symbolically and numerically with the help of a normal mode code, which extremizes the Lagrangian of the system. It is found that the stability limit agrees well with that of the Mercier criterion. There are stable high-beta equilibria in this model.

Introduction

The ideal magnetohydrodynamic (MHD) approximation for describing a plasma is very useful for understanding present tokamak experiments and for designing new devices. In this paper we present results obtained from a computer code which solves for the complete spectrum of normal modes. The method consists in extremizing the Lagrangian of the system connected with linearized perturbations around an equilibrium configuration using a Galerkin procedure leading to a matrix eigenvalue problem [1].

The class of equilibria is characterized by a pressure profile, $p = p_0 - p'\Psi$, and a poloidal current profile,

$$T \equiv X B_\varphi = R[B_0{}^2 + 2\Psi\,\delta\, p'/(1+\alpha^2)]^{1/2},$$

with p_0, p', B_0, δ, α as constants. The flux function Ψ is given by

$$\Psi = p'[Z^2(X^2 - R^2\delta) + \alpha^2(X^2 - R^2)^2/4]/2(1+\alpha^2). \qquad (1)$$

Here R is the radius of the magnetic axis and X, φ, Z denote the usual cylindrical coordinate system. The stability analysis is performed in a non-orthogonal flux coordinate system ψ, φ, θ. For details, we refer to [1–4]. The toroidal current density is

$$j_\varphi = p'X - R^2\delta'/X(1+\alpha^2) \qquad (2)$$

and the (contravariant) poloidal current density j^θ is

$$j^\theta = \tilde{X}\,\alpha\,\delta\,R^2(p')^2/T(1+\alpha^2)^2, \qquad (3)$$

with

$$X = (R^2 + 2\psi\cos\theta)^{1/2},$$
$$\tilde{X} = (X^2 - R^2\delta)^{1/2}. \qquad (4)$$

Near the magnetic axis, $\psi = 0$, the flux surfaces are ellipses with a half-axis ratio

$$e = \alpha/(1-\delta)^{1/2} \qquad (5)$$

and become D-shaped further outward.

The condition that the flux surfaces all be closed within the plasma region imposes the following restrictions:

$$\psi_b/R^2 < 1/2 \quad \text{and} \quad \psi_b/R^2(1-\delta) < 1/2, \qquad (6)$$

where ψ_b denotes the value at the plasma boundary The inverse aspect ratio ε is approximately given by

$$\varepsilon = \psi_b/R^2. \qquad (7)$$

An approximation for the plasma beta, valid for small ε is of the form

$$\beta_T = \frac{1+\alpha^2}{2}\,\frac{\varepsilon^2}{q(0)^2(1-\delta)} \qquad (8)$$

and

$$\beta_p = (1+\alpha^2)/(1+\alpha^2-\delta), \qquad (9)$$

where $q(0)$ denotes the value of the safety factor at the magnetic axis. We emphasize that all configurations have non-vanishing magnetic shear. The magnitude of this shear is given by the inverse aspect ratio and by the poloidal current, i.e. by the parameters ε and δ. Only the cylindrical limit is shearless. Typical q-profiles are shown in Figure 1.

The method and the first application of the code are published in [1]. The main results can be summarized as follows: Toroidal effects are destabilizing for free surface modes, i.e. the conducting wall is far away from the plasma. The stability of internal modes, however, is improved by the toroidicity. Another application of our code has

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Reprint requests to Dr. W. Kerner, Max-Planck-Institut für Plasmaphysik, D-8046 Garching bei München.

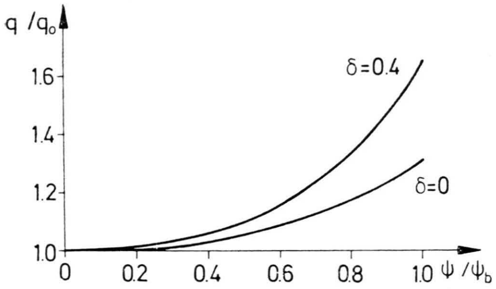


Fig. 1. q profiles for $\varepsilon^{-1} = 4$.

been to provide an independent check of the general Princeton equilibrium and stability code [5] and of the Lausanne code [6]. A careful comparison of the results has given confidence in all three approaches and will be reported elsewhere [7].

In previous publications concerning our equilibrium model, [1, 6—8], only a few configurations, viz. $\delta = 0$ and $\alpha = 1, 2$, or 4, are treated. In this paper we discuss the stability behavior in the full parameter space for modes which leave the plasma boundary unperturbed. We consider only non-axisymmetric perturbations, i.e. $n \neq 0$ in Equation (10). In particular, the connection with the stability limit of a necessary (Mercier) criterion [9, 10] is discussed. Determination of points of marginal stability requires correct evaluation of very small eigenvalues. The strongest restriction on the safety factor from Mercier's criterion is given in the neighborhood of the magnetic axis, see [2]. Therefore, we can use an analytical form, e.g. that of Lortz and Nührenberg [11], which is valid for small distances from the magnetic axis. The value of $q(0)$ necessary for stability is plotted in Fig. 2 as a function of the elongation e for different values of δ.

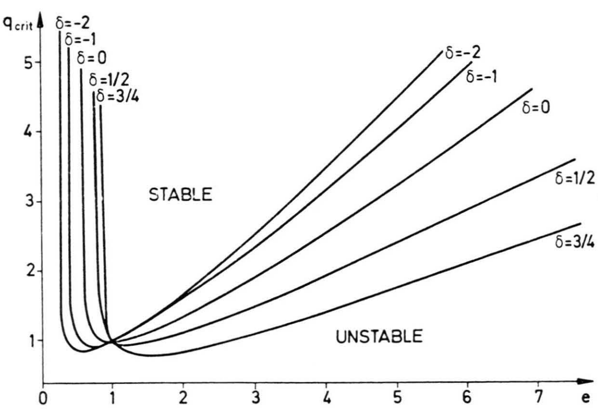


Fig. 2. Mercier criterion near axis.

Accuracy of the Code

A good representation of the spectrum of normal modes is achieved by using a non-orthogonal flux coordinate system ψ, φ, θ and by expanding the displacement vector $\boldsymbol{\xi}$ into modified eigenfunctions of the corresponding straight configurations. By taking advantage of the special analytic equilibrium and with an extended algebraic calculation using the symbolic language REDUCE [12] the mapping and the θ integrations are performed accurately. Integrations over ψ are done with high-order Gaussian quadratures; see also [1], [7].

The normal mode approach has the property that the lowest eigenvalue is always approximated from above as the number of expansion functions is increased. No instability is then found for a stable system. It is our experience that with the set of global Fourier-Bessel expansion functions,

$$\boldsymbol{\xi}(\boldsymbol{r}, t) = e^{-i\omega t} e^{in\varphi} X^2 \tilde{X} \sum_{l,\nu} C_{l,\nu} \boldsymbol{\xi}_{l,\nu}(\psi) e^{il\theta}, \qquad (10)$$

sufficient accuracy is obtained with 9 to 11 Fourier components and 5 or 6 radial expansion functions. An extrapolation to infinitely many expansion functions only changes the eigenvalues by 1 to 8% in most cases. For L Fourier components and M radial functions the eigenvalue ω^2 converges for fixed M as

$$\omega_0{}^2 - C_L \exp(-\alpha_L L),$$

with $\alpha_L \approx 0.5$, and for fixed L as

$$\omega_0{}^2 - C_M \exp(-\alpha_M M),$$

with $\alpha_M \approx 1.0$ and constants α_L, α_M, C_L and C_M.

Figure 3 shows convergence studies for a small aspect ratio case with elliptical cross-section ($\varepsilon^{-1} = 3$, $e = 2$, $\delta = 0$, $q_0 = 0.7$, $q_b = 1.22$). Up to 15 Fourier components and 9 global radial expansion function have been used. In the eigenvalue problem matrices up to a dimension of 270×270 are treated. Such a case needs about 2 minutes CPU time on the Garching IBM 360/91 computer.

Results

In the following discussion we present results for configurations with large aspect ratios, $\varepsilon^{-1} \geqq 10$, and small aspect ratios, $\varepsilon^{-1} = 4$ and 3. Here, n and l denote the toroidal and poloidal wave numbers in Equation (10). We sometimes denote as l_0 the dominant Fourier component in the poloidal

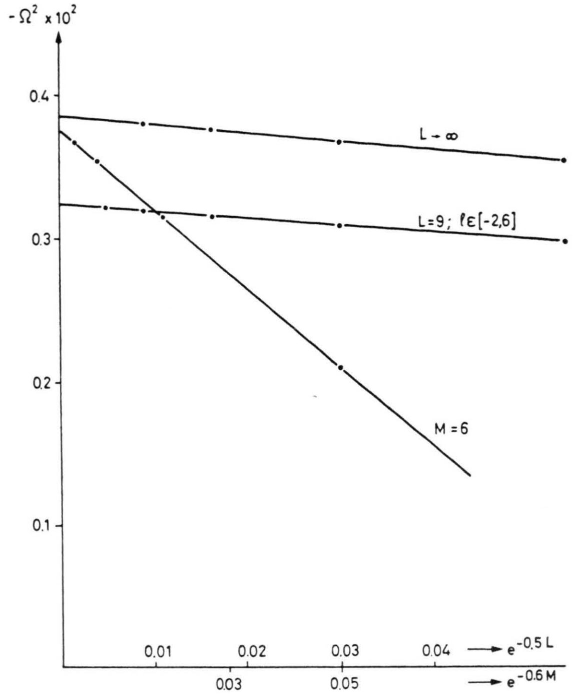


Fig. 3. Convergence studies for $\varepsilon^{-1} = 3$, $e = 2$, $\delta = 0$, $q(0) = 0.7$, $q_b = 1.21$ and $n = 2$. The eigenvalues are plotted versus the number of expansion functions for fixed M with $L = 7, 9, 11, 13$, and for fixed L with $M = 4, 5, 6, 7, 8$. The eigenvalues are extrapolated in L for fixed M and the resulting values are subsequently extrapolated in M, giving the value $\Omega_0{}^2 = -0.386 \cdot 10^{-2}$.

direction θ. The eigenvalues are normalized to the poloidal Alfvèn speed

$$\Omega^2 = \omega^2 \varrho_0 \psi_b{}^2 (1 + \alpha^2)/2 R^2 p_0 (1 - \delta),$$

with a constant plasma density ϱ_0.

A strong MHD instability with $\Omega^2 \approx -1$ corresponds to a time scale of the order of microseconds, a weak instability with $\Omega^2 \approx -10^{-6}$ to milliseconds.

Large Aspect Ratio, $\varepsilon^{-1} \geq 10$

The first interesting result is that for an almost circular cross-section, $e = 1$, no unstable $n = 1$ mode, "internal kink", exists. This holds for all values of α and δ with $e \equiv \alpha/(1-\delta)^{1/2} = 1$. This result is in agreement with [6, 8]. The critical safety factor from the necessary criterion has the value $q(0)_{cr} = 1$. This stability limit is approached by modes with a toroidal wave number $n \to \infty$ and a dominant Fourier component $l_0 = n - 1$, i.e. short wavelength modes. Instabilities then exist for $q(0) \approx l_0/n = (n-1)/n$. We are able to find such unstable modes for $2 \leqq n \leqq 14$, as shown in Figure 4.

The φ and θ integrations for evaluating the matrix elements are done very accurately be means of an extensive algebraic calculation [1, 12]. No difficulty is then caused by the fact that for a case with $n = 14$ and 15 Fourier components centered around $l_0 = 14$ the poloidal wave number l varies between 7 and 21. A different method with a numerical θ integration may, however, require a special procedure for such high l numbers. The eigenvalues for $2 \leqq n \leqq 8$ are quite accurate, at least up to 5%. The range of instability $\Omega^2 = \Omega^2(q)$ in Fig. 4 does not significantly change when more functions are included. For higher n values, $n \geqq 10$, more expansion functions both in θ and ψ are necessary for convergence.

Instead of determining the stability boundary in terms of q for a given elongation, we now determine the critical elongation when $q(0)$ and n are kept fixed. We begin with the fundamental mode $n = 1$. The safety factor is chosen as $q(0) = 1$. The results for the corresponding cylindrical equilibria, [13–15], indicate that the interesting mode should basically be a $n = 1$ and $l_0 = 1$ mode. The growth rates computed as a function of e for values of $\delta = -2, -1, 0, 0.5, 0.75$ are shown in Figure 5a. In Table 1 the computed critical elongations for marginal stability are listed. The two marginal points e_{cr} define an interval in e with stability with respect to the $n = 1$ mode. These values agree well with the localized criterion, [11] and Figure 2. The critical safety factor from the necessary criterion is evalu-

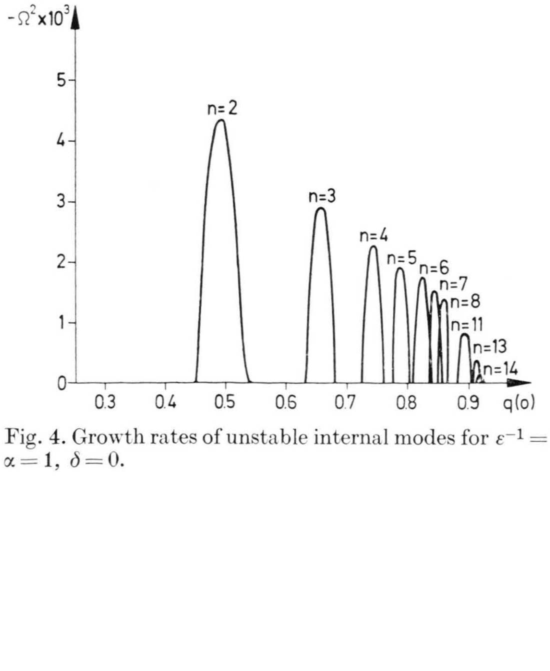
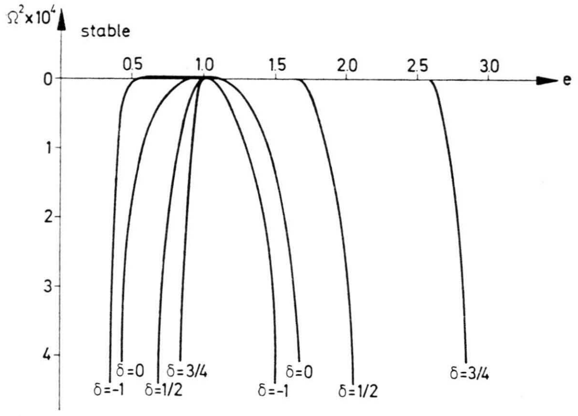
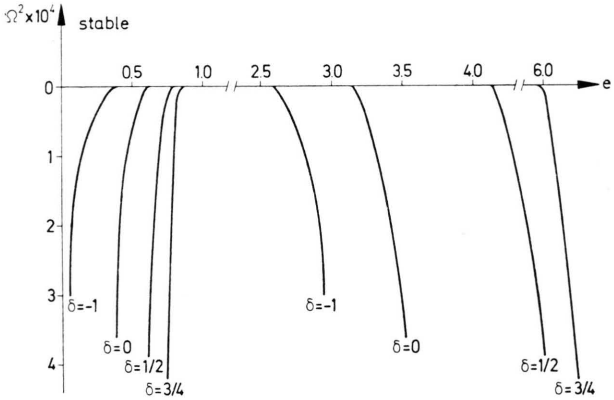


Fig. 4. Growth rates of unstable internal modes for $\varepsilon^{-1} = 10$, $\alpha = 1$, $\delta = 0$.



a)



b)

Fig. 5. Eigenvalues versus ellipticity.
a) $\varepsilon^{-1} = 10$, $n = 1$, $q(0) = 1.0$;
b) $\varepsilon^{-1} = 10$, $n = 1$, $q(0) = 2.0$.

ated for the computed e_{cr} and also listed in Table 1. The agreement with the chosen value $q(0) = 1$ is again very good. It is interesting to note that the stability limit is now given by a long wavelength mode. It is seen from Fig. 5 that the eigenvalues of the unstable modes drop one or two orders of magnitude before the marginal point is reached. The eigenvalues in te stable region are very close to zero since the continuous part of the spectrum extends for $n = 1$, $l_0 = 1$ and $q(0) = 1$ up to the origin with eigenvalues Ω^2 proportional to $(n\,q(\psi_s) - 1)^2$.

At this point we want to discuss the numerical accuracy again. The results stated are obtained with 11 to 15 Fourier components and 5 or 6 radial expansion functions. Increasing the number of expansion further would shift the marginal points

Table 1. Stability limit in comparison with the Mercier criterion.

	$\varepsilon^{-1} = 10$; $n = 1$					
	$q(0) = 1.0$			$q(0) = 2.0$		
δ	e_{cr} (Code)	e_{cr} (Mercier)	q_{cr} (Mercier)	e_{cr} (Code)	e_{cr} (Mercier)	q_{cr} (Mercier)
− 2	0.40	0.40	1.02	0.29	0.29	2.05
	1.01	1.0	1.008	2.42	2.40	2.02
− 1	0.56	0.57	1.017	0.40	0.41	2.02
	1.01	1.0	1.003	2.60	2.57	2.01
0	0.93	1.0	1.005	0.60	0.60	2.02
	1.10	1.0	1.008	3.15	3.10	2.02
1/2	0.99	1.0	1.005	0.78	0.77	2.02
	1.66	1.63	1.003	4.16	4.08	2.03
3/4	0.99	1.0	1.02	0.86	0.88	2.10
	2.60	2.52	1.02	5.92	5.63	2.10

only a little to even better agreement with the necessary criterion. In particular, the inclusion of more radial functions will produce stable modes with growth rates closer to zero.

In order to discuss further details, it is instructive to look at the corresponding eigenfunctions. The eigenfunctions of the dominant Fourier component $l_0 = 1$ for $n = 1$ and $l_0 = 2$ for $n = 2$, are plotted in Figs. 6 and 7 for a case with $\delta = -1$. Here $\xi^1(\psi) = R\,\xi^\psi$ and $\xi^3(\psi) = \psi\,\xi^\theta/R$ denote contravariant components. The ξ^φ component is too small to be shown. The contribution of the other Fourier components is at least one order of magnitude

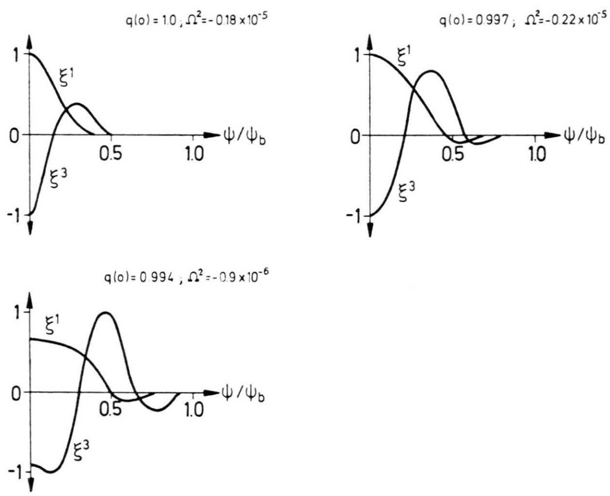


Fig. 6. Eigenfunctions for $n = 1$, $l_0 = 1$ modes $\xi^1 \equiv R\,\xi^\psi$, $\xi^3 = \psi\,\xi^\theta/R$, $\varepsilon^{-1} = 10$, $\delta = -1$, $e = 0.53$.
a) $q(0) = 1.0$, $\Omega^2 = -0.18 \cdot 10^{-5}$;
b) $q(0) = 0.997$, $\Omega^2 = -0.22 \cdot 10^{-5}$;
c) $q(0) = 0.994$, $\Omega^2 = -0.9 \cdot 10^{-6}$.

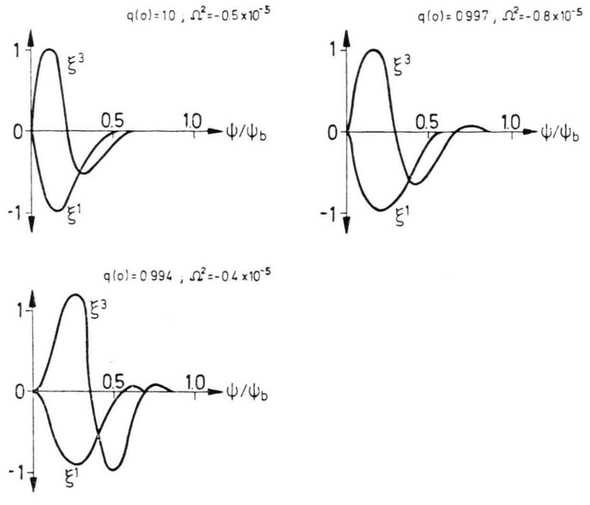


Fig. 7. Eigenfunctions for $n = 2$, $l_0 = 2$ modes $\xi^1 \equiv R\,\xi^\psi$, $\xi^3 = \psi\,\xi^\theta/R$, $\varepsilon^{-1} = 10$, $\delta = -1$, $e = 0.53$.
a) $q(0) = 1.0$, $\Omega^2 = -0.5 \cdot 10^{-5}$;
b) $q(0) = 0.997$, $\Omega^2 = -0.8 \cdot 10^{-5}$;
c) $q(0) = 0.994$, $\Omega^2 = -0.4 \cdot 10^{-5}$.

smaller, with the exception of the $l_0 \pm 1$ components. Here only the toroidal component ξ^φ is present. This coupling is due to toroidal effects and can be explained by studying the spectrum at the origin, [16]. Assuming $|\xi^\psi| \ll |\partial/\partial_\psi\, \xi^\psi|$, the marginal points of the potential energy are defined by the following equations (see also [4]):

$$n\,q - l_0 = 0\,, \tag{11}$$

$$\nabla \cdot \boldsymbol{\xi} = 0\,, \tag{12}$$

$$\nabla \cdot \boldsymbol{\xi}_\perp / X^2 = 0\,. \tag{13}$$

Here $\boldsymbol{\xi}_\perp$ denotes the part of the displacement vector perpendicular to the magnetic field.

Combining Eqs. (12) and (13) and using Eq. (4) leads to

$$\frac{\partial}{\partial \psi} X^2 = 2 \cos\theta\,, \quad \frac{\partial}{\partial \theta} X^2 = -\,2\,\psi \sin\theta\,.$$

The marginal point for the perturbed toroidal field, Q_φ, then yields the condition

$$\sum_l e^{il\theta}[(n\,q - l)\,\xi_l{}^\varphi + F_{l+1}^- + F_{l-1}^+] = 0, \tag{14}$$

with $F_l^\mp = \xi_l{}^\psi \pm \psi\,\xi_l{}^\theta$.

Evaluating Eq. (14) shows that $\xi^\varphi_{l_0\pm1}$ is of the order of $\xi^\psi_{l_0}$ and $(\psi\,\xi^\theta_{l_0})$, and that $\xi^\psi_{l_0\pm1}$ and $(\psi\,\xi^\theta_{l_0\pm1})$ are small and hence $\xi^\varphi_{l_0}$ is as well.

It is seen from Fig. 6, 7 that the eigenfunctions are gross modes similar to the straight system eigenfunctions, but vanish in the outer part of the plasma, where the shear is strongest. If $q(0)$ is chosen smaller than 1.0, the eigenfunctions now extend in the radial direction up to the plasma boundary, as can be seen from Figs. 6 and 7. More then one instability, corresponding to eigenfunctions with several radial nodes, exist. The shape of the eigenfunctions does not depend much on whether e is close to e_{cr} or not. Generally, instabilities exist for a mode with wave numbers n and l_0 if $n\,q(0)$ has values between $l_0 - \eta_1$ and $l_0 + \eta_2$ with constants η_1, η_2 $(\eta_1 \neq \eta_2)$. But, if for an unstable mode the elongation e approaches the value e_{cr}, the interval in $q(0)$ for which instabilities occur shrinks to one point $q(0) = n/l_0$. For $n = 1$ and $l_0 = 1$ this point is $q(0) = 1$. This is shown in Figure 8. According to Mercier [17] such a configuration with $e = 1$ is marginally stable for $q(0) = 1$. The numerical evaluation of the Mercier criterion for finite ε even indicates stability for $q(0) = 1$. Hence, the considered $n = 1$, $n_0 = 1$ mode, would be unstable in a Mercier stable region. But such an instability is not found.

Pao [18] has shown that a $n = 1$ and $l_0 = 1$ instability of a circular cylinder should also occur in the corresponding toroidal configurations. Because of the weak shear near the axis in our equilibrium model the singular region, which is assumed to be small in [18], is here of considerable radial extent and so this analysis cannot be applied here.

The region of stability with respect to the $n = 1$ mode can be enlarged by a suitable choice of the

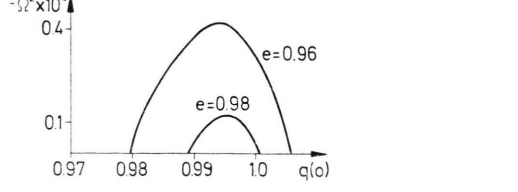


Fig. 8. Eigenvalues versus safety factor $\varepsilon^{-1} = 10$.
a) $n = 1$, $\delta = -1$;
b) $n = 2$, $\delta = -1$;
c) $n = 1$, $\delta = 0.75$.

poloidal current. For a paramagnetic current distribution, $\delta < 0$, the stable region occurs for $e < 1$, i.e. lying ellipses, and for a diamagnetic one, $\delta > 0$, for standing ellipses. This stabilizing effect may be important for Belt Pinch devices with a possible increase in the plasma beta by a factor of 5 to 10 compared with the circular cross-section. The results from a similar analysis for $n = 1$ and $q(0) = 2.0$ are shown in Fig. 5b and Table 1. Note that for elliptical cross-sections more Fourier terms are necessary for convergence.

Finally, we can summarize the results concerning stability: The stability limit agrees well with that of the Mercier criterion for all values of α and δ. If for a chosen $q(0) = l_0/n$ the necessary criterion is violated there, is an unstable mode with a wave number n and $nq(0) - l_0 \approx 0$. This result has been obtained for different values of n with n varying between one and five and in some cases up to 15.

We have not found any unstable mode, e.g. a ballooning type mode [19—21], in the Mercier stable domain of parameter space. Our results indicate that such modes — if they exist — have very high wave numbers n, l, i.e. n much larger than 10, or very small growthrates, i.e.

$$|\Omega^2| \ll 10^{-6} \propto \varepsilon^6 .$$

Small Aspect Ratio, $\varepsilon^{-1} \approx 4$ or 3

It is an important question whether additional instabilities occur if the aspect ratio is decreased. It turns out that toroidicity improves the stability behavior.

We again examine the $n = 1$ mode. For an almost circular cross-section, $e = 1$, this mode remains stable for small aspect ratio. For $q(0) = 0.99$ the critical elongation for marginal stability is listed in Table 2. These values are very similar to the large aspect ratio cases with increased regions of stability.

We discuss in detail the stability of a JET-like configuration, $e = 2$, with two different values of β_p. The aspect ratio chosen is $\varepsilon^{-1} = 4$. For smaller values of ε^{-1} the parameter δ, restricted by $\delta < 1 - 2\varepsilon$, is essentially zero and hence $\beta_p = 1$.

The two cases differ in the stability boundary of the necessary criterion, $q_{cr} = 1.37$ for $\delta = 0$ and $q_{cr} = 1.18$ for $\delta = 0.4$. The q profiles are plotted in Figure 1. The growth rates of the unstable modes with $n = 1, 2, 3,$ and 4 are plotted in Figure 9.

Table 2. Stability limit in comparison with the Mercier criterion $n = 1$, $q(0) = 0.99$.

ε^{-1}	δ	e_{cr}(Code)	e_{cr}(Mercier)	q_{cr}(Mercier)
	0	0.91	1.0	1.01
		1.09	1.0	1.01
4	0.4	0.97	1.0	1.01
		1.48	1.44	1.01
	0	0.90	1.0	1.01
		1.11	1.0	1.01
3	0.25	0.94	1.0	1.01
		1.30	1.23	1.02

For configurations with $e = 2$ and $q(0) = 1.0$ the Mercier criterion is violated. Therefore, the $n = 1$ and $l_0 = 1$ mode is unstable for both cases. For a toroidal wave number $n = 2$ the $l_0 \geqq 3$ modes with $q(0) \approx l_0/n \geqq 1.5$ are Mercier stable and, hence, these modes are found to be stable by the code. But the $l_0 = 1$ and 2 modes are unstable and strongly coupled. Owing to the larger shear the $l_0 = 1$ mode is shifted to smaller values of $q(0)$ for $\delta = 0.4$ in comparison with $\delta = 0$.

For $n = 3$ and $l_0 = 4$ the safety factor $q(0) \approx 4/3 = 1.33$ violates the necessary criterion for $\delta = 0$ but satisfies it for $\delta = 0.4$. Hence the $l_0 = 4$ mode is unstable in the case with $\delta = 0$, but stable when $\delta = 0.4$. The $l_0 = 3,2$ and 1 modes are unstable in both cases. For modes with $n = 4$ the results are similar. There are JET-like equilibria which are stable with respect to internal modes and have a plasma beta of the order of 12%.

Summary

The MHD stability is discussed for a class of analytic tokamak equilibria with essentially flat volume current profiles but with elliptical or D-shaped cross-sections. The normal modes of the system are computed using a Ritz-Galerkin procedure in the Lagrangian formalism. The equilibria of this model are highly unstable with respect to external kink perturbations. But, at least theoretically, these instabilities can be stabilized by a perfectly conducting wall placed on the plasma surface. With our method small eigenvalues can be evaluated correctly and the points of marginal stability can be determined precisely.

The basic result is that the stability limit for a fixed plasma boundary agrees with that of the

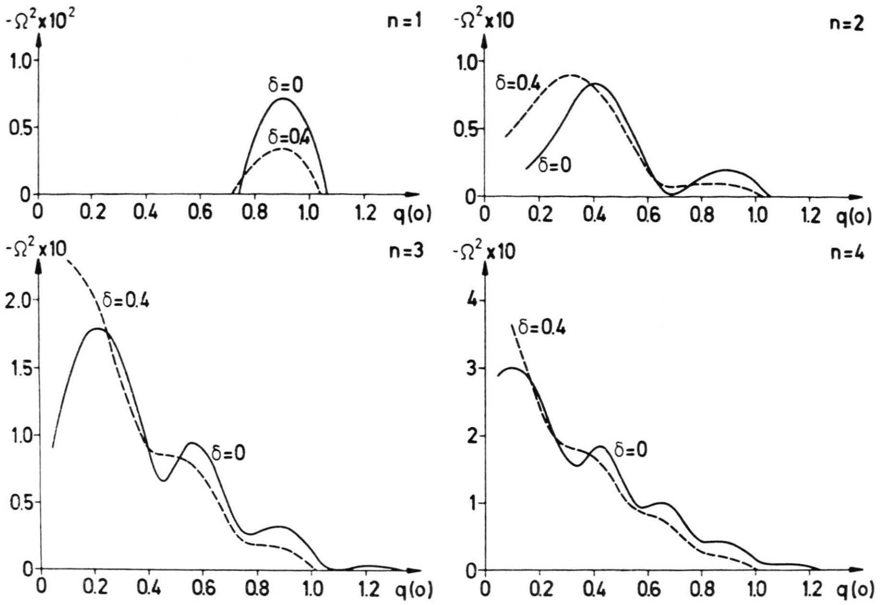


Fig. 9. Growth rates versus safety factor $\varepsilon^{-1} = 4$, $e = 2$.

Mercier criterion for this entire class of tokamak equilibria. For a circular cross-section, $e = 1$, there is no unstable $n = 1$ mode, "internal kink", and the stability limit is given by short-wavelength modes with large n and l_0. For shaped cross-sections the stability limit is also reached by $n = 1$ modes.

The equilibria considered here have a poloidal plasma beta, which cannot be increased arbitrarily but is considerably smaller then the aspect ratio. This fact may explain the absence of ballooning type modes. We conclude that such modes — if they exist — have very high wave numbers.

The study of other equilibria with peaked current profiles is planned for the future. We expect that a stabilizing effect on the external kink instabilities is connected with more internal instabilities. An interesting question then is whether the flattening of the current profile in some areas, e.g. around $q = 2$ and $q = 3$ surfaces, again has a benefical effect on the internal modes.

[1] W. Kerner, Nucl. Fus. **16**, 643 (1976).
[2] G. Küppers, D. Pfirsch, and H. Tasso, in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 4th Int. Conf. Madison, 1971) **2**, IAEA, Vienna (1971) 529.
[3] W. Kerner and H. Tasso, in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 5th Int. Conf. Tokyo, 1974) **1**, IAEA, Vienna 1975, 475.
[4] W. Kerner, IPP 6/134, Garching, March 1975.
[5] R. C. Grimm, J. M. Greene, and J. L. Johnson, in "Methods in Computational Physics", Vol. **16**, Academic Press, New York 1976, 253.
[6] D. Berger, L. C. Bernhard, R. Gruber, and F. Troyon, in Plasma Physics and Controlled Nuclear Fusion 1976, IAEA, Vienna (1977) Paper CN-35/B 11-4 (1976).
[7] M. S. Chance, J. M. Greene, R. C. Grimm, J. L. Johnson, J. Manickam, W. Kerner, D. Berger, R. Gruber, and F. Troyon, to be published in J. Comp. Phys.
[8] A. Sykes and J. A. Wesson, in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 5th Int. Conf. Tokyo 1974) **1**, Vienna 1975, 449.
[9] C. Mercier, Nucl. Fusion **1**, 47 (1960).
[10] J. M. Greene and J. L. Johnson, Phys. Fluids **5** (1962), 510.
[11] D. Lortz and J. Nührenberg, Nucl. Fusion **13**, 821 (1973).
[12] W. Kerner and J. Steuerwald, J. Comput. Phys. Comm. **9**, 337 (1975).
[13] R. J. Taylor, Proc. Phys. Soc. London **B 70**, 1043 (1957).
[14] M. S. Chance, J. M. Greene, R. C. Grimm, and J. L. Johnson, Nucl. Fusion **17**, 53 (1976).
[15] D. Berger, R. Gruber, and F. Troyon, Comp. Phys. Communications **11**, 313 (1976).
[16] R. Gruber, private communication.
[17] C. Mercier, Nucl. Fusion **4**, 213 (1964).
[18] Y. P. Pao, Phys. Fluids **19**, 1796 (1976).
[19] A. M. Todd, M. S. Chance, J. M. Greene, R. C. Grimm, J. L. Johnson, and J. Manickam, Phys. Rev. Letters **38**, 826 (1977).
[20] D. Dobrott, D. B. Nelson, J. M. Greene, A. H. Glasser, M. S. Chance, and E. H. Frieman, Phys. Rev. Letters **39**, 943 (1977).
[21] J. M. Connor, R. J. Hastie, and J. B. Taylor, Phys. Rev. Letters **40**, 396 (1978).